参赛说明

1．在赛前我们用 E-mail，并书面通知了每个学校的参赛队号，如果仍有学校没有收到队号，请于竞赛期间尽快与组委会联系，在交卷时一定要加上指定的队号，并另寄报名表（不要夹在答卷中）。

2．如果在竞赛中对赛题的理解方面有问题，可以在 www.shumo.com 的论坛上提问，我们将组织命题人在网上解答，不应回答的，恕不回答。

3．由于是研究生的竞赛题，有一定的难度，因此不必做完上一个问题，才能回答下一个问题。而且为了完整地把实际问题表达出来，题目中的问题较多，很可能在四天之内做不完，因此对后面的问题也可以不作回答，有兴趣的同志可以在竞赛后再作深入研究。

4．每队在 20 日 10：00 前用特快专递寄出论文（以当地邮戳为准）。并请于赛后几小时之内再将电子版的论文（用光盘或软盘，可以一个学校的论文刻在一张光盘上）寄给组委会，注意一定与纸质论文分开来寄。

5．由于与大学生数学建模竞赛时间相同，而评审委员多数身兼两职，因此评审时间可能较晚，敬请广大参赛研究生谅解。在此期间欢迎大家对赛题进行深入探讨，我们准备和 2004 年一样正式发表研究生竞赛的优秀论文，还准备正式发表一些通过竞赛之后的讨论写出的有关赛题的优秀论文，欢迎广大师生积极参与。

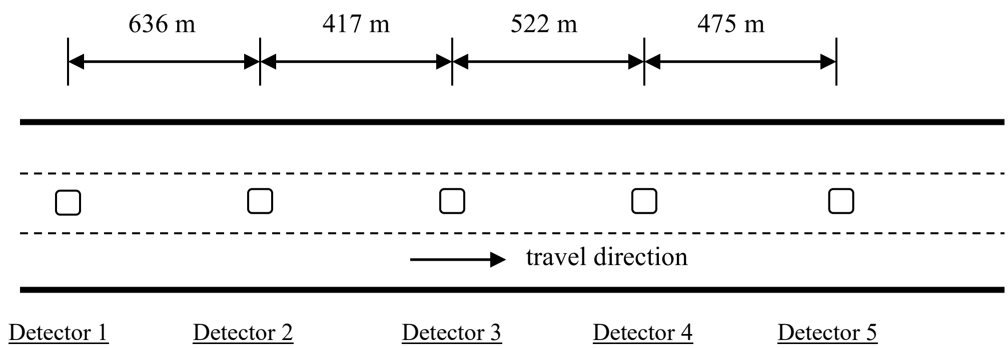
6．由于题目难度不可能完全相同，评审中将向难度较大的题目倾斜，请研究生在选题时加以考虑。

2005年全国部分高校研究生数学建模竞赛 A 题

A: Highway Traveling time Estimate and Optimal Routing

I

Highway traveling time estimate is crucial to travelers. Hence, detectors are mounted on some of the US highways. For instance, detectors are mounted on every two-way six-lane highways of San Antonio city. However, since vehicles tend to change lanes from time to time, we may ignore vehicle lane change and consider just one lane traffic as shown below, in which the square boxes stand for detectors.



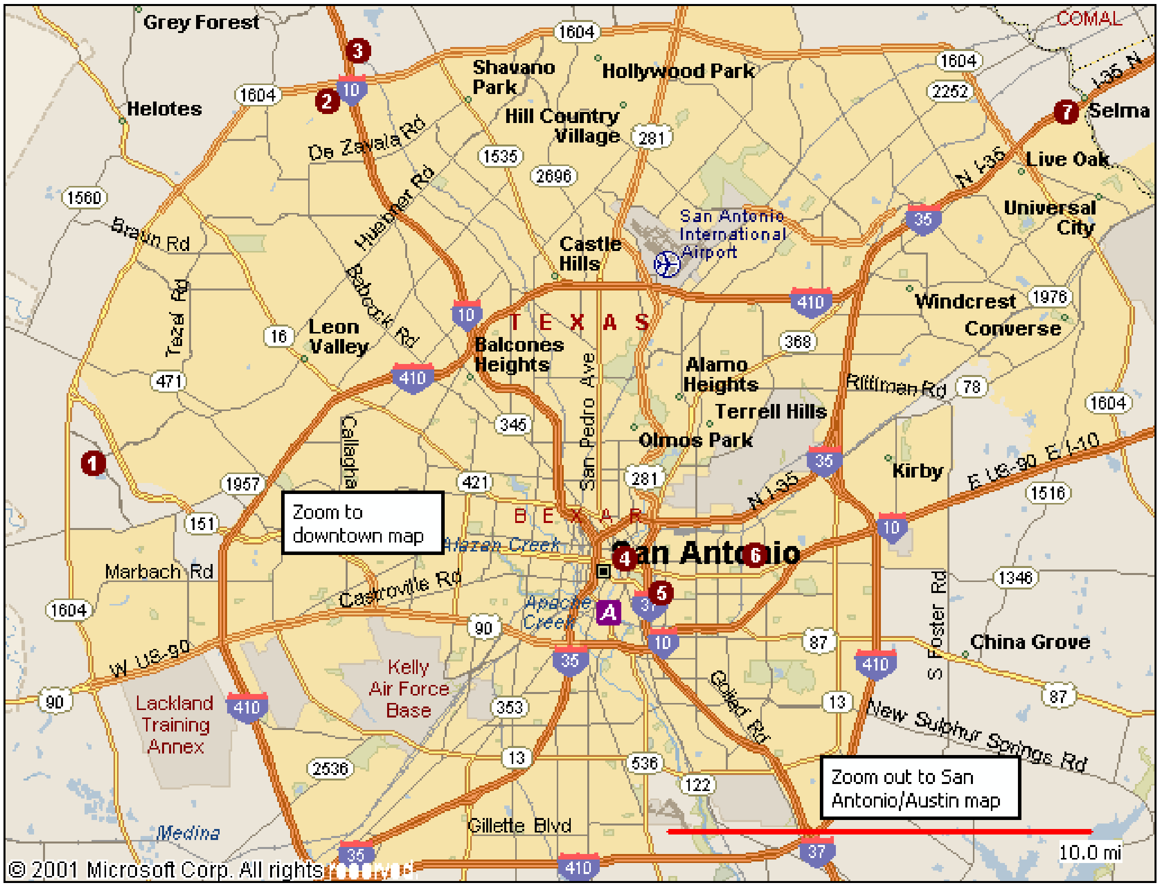
1. Detectors are able to detect and measure the speed of individual vehicles 24 hours a day. Average vehicle speed measured within 20 seconds by detectors is reported and refreshed. The following table provides the real time data (due to the huge volume of traffic data, only traffic data of the last 20 seconds in every two minutes is in the last 20 seconds. Unit: mile/hour). Please analyze traveling characteristics on highways (for instance, congestion and its dispersion. Typically, it is not considered as congestion if traffic speed is higher than 50 mile/hr). If a vehicle passes the detector at time t, how long will it take for this vehicle to travel to the fifth sensor? Please design an algorithm for estimating such travel times. Make sure that you demonstrate the rationality and accuracy of your algorithm. If traffic data is provided every 20 seconds rather than every 2 minutes, how this information is going to affect your estimate?

All the conditions stay the same as in the previous problem. If detectors can measure not only vehicle speed, but also traffic volume per unit time (see table below. The unit of Flow is the number of cars/20 seconds), does this additional information help to improve the rationality and accuracy of your algorithm? If your answer is yes, please re-design your algorithm.

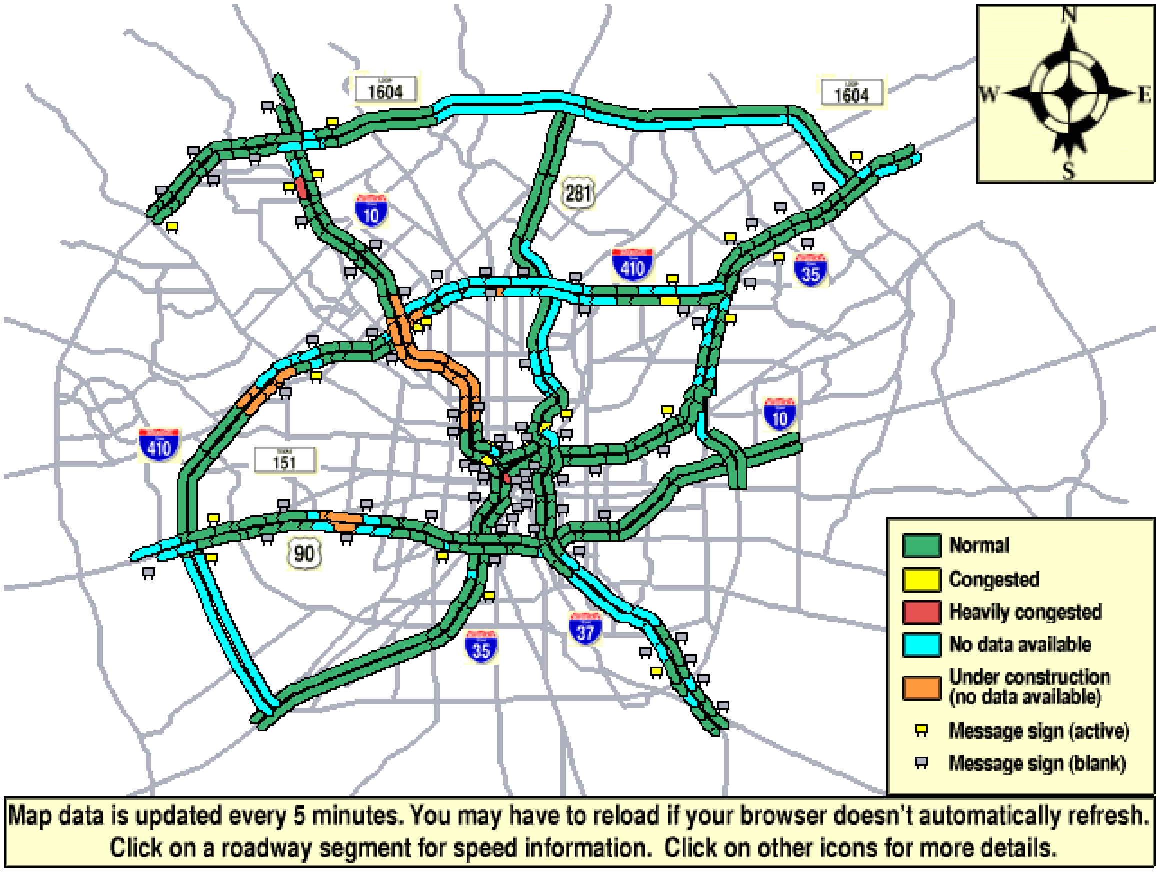
Time	Sensor 1		Sensor 2		Sensor 3		Sensor 4		Sensor 5	
	Speed	Flow	Speed	Flow	Speed	Flow	Speed	Flow	Speed	Flow
3:40:07 PM	57	10	54	9.7	62	8.9	20	10.7	58	5.9
3:42:07 PM	62	9.5	68	11.4	63	13.6	21	10	59	12.2
3:44:07 PM	56	11.1	62	10.3	61	12.9	19	14.2	60	8.1
3:46:07 PM	58	12.5	61	9.1	59	9.3	23	14.8	61	10
3:48:07 PM	53	11.8	64	6.7	62	12.3	45	9	59	10.1

3:50:07 PM	58	9.3	66	6.4	63	4.5	63	10	56	13.8
3:52:07 PM	55	11.9	63	13	60	9.2	55	11.4	59	18
3:54:07 PM	59	8	62	5.3	63	9.7	65	11	55	11
3:56:07 PM	52	9.8	73	9.7	56	8.3	65	8	62	12
3:58:07 PM	60	14.1	63	13.7	59	12	66	5.1	61	5
4:00:07 PM	55	4.8	64	12.3	61	12.8	65	5.1	59	11
4:02:07 PM	60	13.7	61	7.9	60	12.6	65	11.1	66	0
4:04:07 PM	59	8.7	62	9.1	60	4.1	64	10.6	57	14
4:06:07 PM	58	8	64	8.3	64	14.5	45	7	59	14
4:08:07 PM	55	5.4	63	15.3	63	16	1	9.3	57	7
4:10:07 PM	62	13.7	61	9.1	57	5.4	15	11.5	64	8
4:12:07 PM	59	9.9	63	6.7	62	5	64	6.1	52	14
4:14:07 PM	58	8.5	63	12	62	11.9	40	8.7	60	10
4:16:07 PM	56	8.3	58	9	60	13.7	61	11.5	58	13
4:18:07 PM	60	9.3	64	10.3	65	10	52	10	51	9
4:20:07 PM	59	10.4	66	9.9	68	5	45	14.7	55	15.9
4:22:07 PM	53	14	56	8.6	63	8	55	13.7	40	12.7
4:24:07 PM	57	13	62	13.5	62	13	29	14	54	11.9
4:26:07 PM	58	13.4	64	11.5	63	14	62	13	30	12.9
4:28:07 PM	55	8.3	62	11.3	58	12	8	10.9	33	11.9
4:30:07 PM	59	11	78	8	61	13	39	5.1	51	10.1
4:32:07 PM	59	17.1	67	11.8	61	11	57	11.9	59	16.9
4:34:07 PM	54	7.3	64	10.6	59	9	62	10	60	11.1
4:36:07 PM	59	12	66	7.8	68	5.1	62	10	27	8.3
4:38:07 PM	56	11.7	68	10.4	60	8	20	8.2	49	13.9
4:40:07 PM	57	8.4	60	11.2	65	14.9	37	11	51	11.1
4:42:07 PM	55	10.4	63	12.3	66	6	40	9.9	45	14.2
4:44:07 PM	59	12.1	65	9.2	62	6.9	49	8.8	51	14.6
4:46:07 PM	49	13.4	69	11.5	66	8.1	54	7.8	20	9.2
4:48:07 PM	51	11.8	66	14	61	10.2	21	14.4	30	12.3
4:50:07 PM	53	11.6	60	14.7	62	10.1	2	5.5	34	11.9
4:52:07 PM	53	13.8	62	12.2	50	9.2	21	12	36	7.5
4:54:07 PM	49	14.3	59	12.7	59	10.1	5	3.8	38	14.8
4:56:07 PM	53	13.5	62	13.6	56	11	38	13.2	26	10.1
4:58:07 PM	55	12.8	62	12.8	57	11	23	8.2	37	13.8
5:00:07 PM	56	11.9	62	13	57	10.1	4	14.8	34	11
5:02:07 PM	56	7.6	63	9.7	59	8.9	2	11.9	28	11.6
5:04:07 PM	60	6.6	65	14.7	64	14	47	9.1	29	10.8
5:06:07 PM	55	11.9	63	11	66	7.6	36	12.9	28	9.8
5:08:07 PM	59	10.2	65	8	60	9	40	12.1	37	14.3
5:10:07 PM	60	8.2	64	11	66	9.2	26	14.8	29	12.2
5:12:07 PM	60	8.7	64	5.6	64	7.5	13	9.4	38	13.2
5:14:07 PM	55	13.1	62	8.4	66	5.9	63	8.2	34	12
5:16:07 PM	56	15.5	57	12.4	61	10	61	10.8	27	13
5:18:07 PM	56	13.3	63	5.5	64	6.1	23	7.9	17	9.4
5:20:07 PM	57	13.1	57	12	65	11.9	4	8.8	14	10
5:22:07 PM	51	14	60	10.9	59	7.5	4	9.2	19	6.4
5:24:07 PM	48	12.5	59	8.9	27	9	17	8	21	9
5:26:07 PM	46	11.6	57	7.4	2	8.9	5	7.8	28	9.7
5:28:07 PM	52	12.4	53	12	26	8.9	3	8.5	26	7.9
5:30:07 PM	57	12.6	42	10.6	12	7.8	3	8.5	18	8.2
5:32:07 PM	51	14.4	38	11.6	10	8.5	2	7.9	19	7.8
5:34:07 PM	51	11.4	19	4.7	21	9.2	22	8.3	33	5.6

5:36:07 PM	53	12.4	14	3.7	12	5	2	6.8	42	9.7
5:38:07 PM	40	10.2	13	8.3	2	7.5	3	7.4	40	10.3
5:40:07 PM	39	7.9	14	6.4	2	7.5	2	9.3	38	9.6
5:42:07 PM	42	10.4	8	7	2	8.3	2	9.7	44	9.4
5:44:07 PM	46	5.8	4	6.4	2	5.6	3	7.7	38	11.3
5:46:07 PM	44	6.9	8	5.3	9	8.1	5	9.7	47	10.7
5:48:07 PM	38	8.3	7	6.7	7	7.8	12	11.3	24	11.1
5:50:07 PM	45	8.6	4	7.7	22	10	3	10	29	8.3
5:52:07 PM	42	9	35	9.4	27	11.1	4	8.7	25	8.1
5:54:07 PM	42	7.8	38	7.4	3	5.4	22	11	11	8.7
5:56:07 PM	43	8.5	56	8.6	26	9.7	4	9	16	6.1
5:58:07 PM	45	9.4	59	6.3	23	8.3	1	9	15	7.7
6:00:07 PM	42	10	63	8.7	42	8	9	8.6	20	9.6
6:02:07 PM	40	10.4	62	10.5	38	9	20	6.5	26	7
6:04:07 PM	43	9.7	61	8.8	14	4.4	1	7.1	25	6.6
6:06:07 PM	42	10.3	59	9.9	21	6.1	2	8	11	9.4
6:08:07 PM	41	10	59	9.2	8	6.4	11	7.9	10	7.8
6:10:07 PM	35	9.4	66	9.5	29	9.7	2	7.6	14	8.4
6:12:07 PM	53	5.7	60	10	39	10.8	5	8.6	23	10
6:14:07 PM	57	4	65	10.5	39	8.8	1	9.2	11	7.6
6:16:07 PM	52	7.3	64	8.5	55	8.6	3	9.6	4	10.6
6:18:07 PM	54	3.3	66	4.6	51	6.2	2	9.6	4	11
6:20:07 PM	55	10.6	62	6.9	62	4.8	4	8.9	3	9
6:22:07 PM	53	2	64	4.8	60	7.2	5	4.9	4	9.9
6:24:07 PM	61	7.5	66	12.1	62	11.4	72	6.2	4	9.5
6:26:07 PM	58	6.7	65	6.7	61	3.8	63	8.8	4	9.6
6:28:07 PM	57	6.1	69	6.6	67	4	45	7.5	12	8.9
6:30:07 PM	58	6.3	64	6.2	62	6.4	34	5.2	36	7.3
6:32:07 PM	57	5.3	66	4.8	61	6.7	58	6.4	24	10.5
6:34:07 PM	58	9	62	4.4	69	6.4	45	5.7	53	12.5
6:36:07 PM	59	4.4	71	2.4	70	5.8	15	8.9	63	6
6:38:07 PM	58	4	64	4.9	64	4.1	9	8.2	65	6
6:40:07 PM	60	4.6	70	0.6	68	2	6	7.6	59	5.4
6:42:07 PM	60	6.4	62	4.6	64	3.7	28	6.9	64	5.5
6:44:07 PM	57	1.6	65	8	66	7.6	44	6.6	59	4.1
6:46:07 PM	55	8.8	64	3.6	72	4.1	55	6.2	62	6.6
6:48:07 PM	58	5.7	66	9.4	62	6.5	40	3.2	64	2.9
6:50:07 PM	59	6.2	62	2.4	67	6	52	0.5	65	7
6:52:07 PM	61	7.3	66	3.1	66	4	58	1.2	66	7.7
6:54:07 PM	59	6.7	69	6.1	64	7	64	1.9	64	8.8
6:56:07 PM	64	3.5	69	4.8	62	7	59	5.3	62	6.3
6:58:07 PM	64	4.2	68	3.9	57	4.5	36	4.4	64	4.1



Grey Forest
Helotes
Shavano Park
Hollywood Park
Hill Country Village
Selma
Live Oak
Universal City
San Antonio International Airport
Castle Hills
Windcrest
Converse
Leon Valley
Balcones Heights
T E X A S
Alamo Heights
Terrell Hills
Olmos Park
Kirby
B E X A R
San Antonio
China Grove
Kelly Air Force Base
Lackland Training Annex
Medina
COMAL
De Zavala Rd
Huebner Rd
Babcock Rd
Braun Rd
Tezel Rd
Callaghan
San Pedro Ave
Rittiman Rd
N I-35
I-35 N
E US-90 E I-10
Marbach Rd
Castroville Rd
W US-90
Alazan Creek
Apache Creek
Goliad Rd
S Foster Rd
New Sulphur Springs Rd
Gillette Blvd
Zoom to downtown map
Zoom out to San Antonio/Austin map
10.0 mi



II

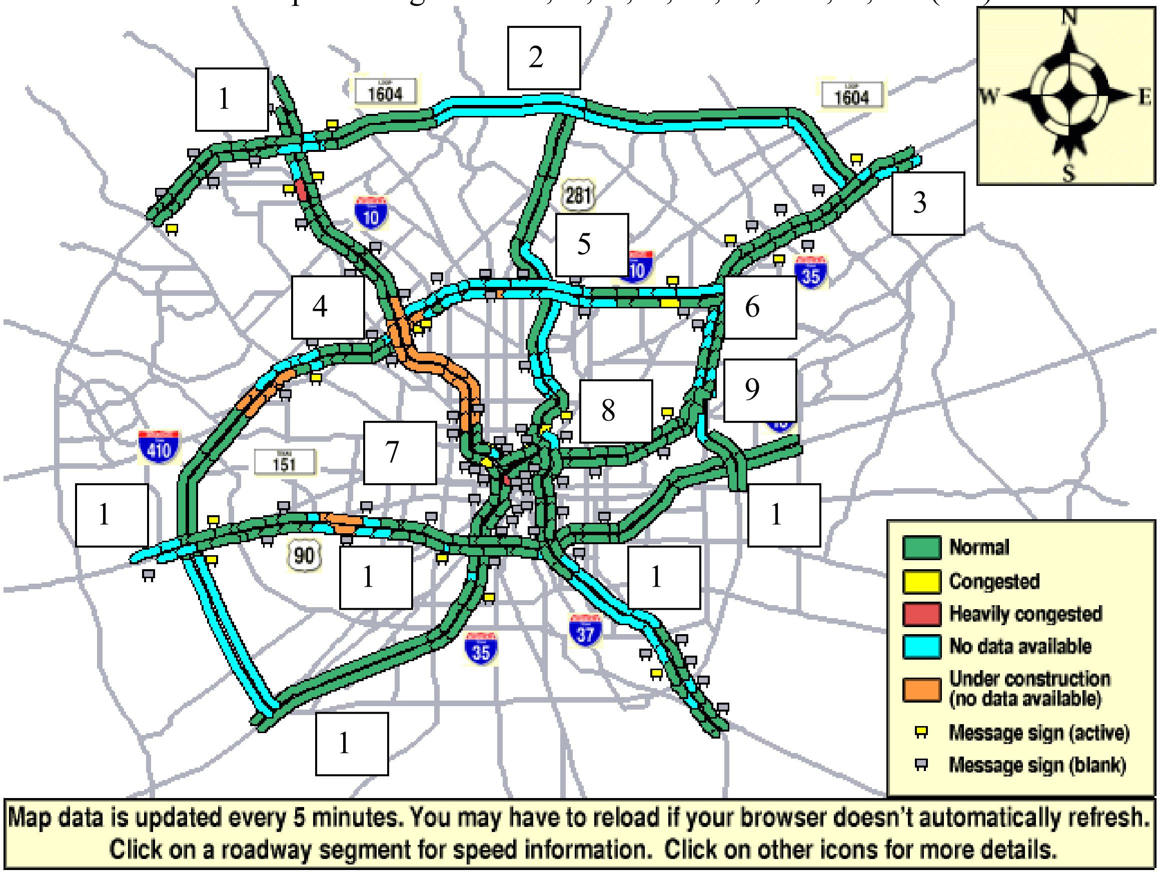
The first figure is a city map of San Antonio in Texas, the United States. The second one is a map reflecting traffic condition in San Antonio. Travelers can input their locations and destinations into an on-board in-vehicle navigation system so that the system will help to select a driving route and estimate the traveling time. Unfortunately, due to the uncertainty in traffic time of each link (road between two nodes, nodes can be treated as intersections), the existing system has a poor performance in terms of providing an optimal (fastest) route and a reliable travel time estimate.

Can you improve the system based on question 1?

1. Provided link travel times are mutually independent random variables. Please design an algorithm for the system to address optimal route selection and travel time estimation. Make sure to clarify your definition of optimal route.

2. Travel times on individual links are dependent on departure time, as well as are mutually correlated. A time dependent covariance matrix Cov(ij,kl) is typically used to quantify such a time-varying correlation relationship, in which i, j, k, and l represent the two nodes of two links. Use the maps above to design a rational matrix and design an algorithm for optimal routing. Be sure to clarify your definition of the optimal route. If

there are n nodes, then the time dependent function Cov(ij,kl) is a n(n-1)/2 sqaure matrix, each row or column representing a link 12,…,1n,21, …,2n,…n1,n2,… n(n-1).



III

1-----2	8. 1
2-----3	9. 15
1-----4	6. 9
2-----5	5. 85
3-----6	5. 25
4-----5	5. 1
5-----6	4. 95
4-----10	9. 6
4-----7	6. 15
7-----8	1. 95
5-----8	6
8-----9	5. 1
6-----9	3. 15
10----11	8. 55
7-----11	2. 55

8-----12	2.85
11-----12	2.4
9------13	2.25
12-----13	6.15
10-----14	5.55
11-----14	8.1

From the above figure, two bold lines in two different directions designate highways with the upper line going from left to right and left line from top down. Vehicles passing through the intersections (nodes) are able to reach other links connected with these nodes. There are fourteen nodes in the above map. The distances of every two nodes are listed in the above table. Find an optimal route from node 3 to node 14, and from node 14 to node 3, respectively. Also, please provide a travel time estimate for each route. It is assumed that travel time follows the same rule as question 1, average link travel time is proportional to link length, variance of link travel time is proportional to the reciprocal of $(link\ length)^{2/3}$ as well as proportional to the product of the number of links connecting to the two nodes at both ends of the link.